### BAYESIAN SURVEILLANCE FOR DETECTION OF SMALL AREA HEALTH ANOMALIES

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# BACKGROUND

Bayesian modeling

The surveillance task

> Bayesian modeling of spatio-temporal health data

- $\geq$  Risk models
- > Model fitting: MCMC and INLA
- Prospective fitting issues

## **BAYESIAN MODELING**

Bayesian models consist of two components:

- Likelihood for the data
- Prior distributions for the parameters

These are combined to form a *posterior distribution* for the parameters

$\Pr(parameters   data) \propto$	L(data   parameters)	). Pr(parameters)
posterior	likelihood	priors

### THE SURVEILLANCE TASK

Public health surveillance is the focus

Health data usually consist of aggregated counts of disease within small areas (counties, districts, postal codes,...)

Surveillance is essentially about change

> There are a number of things we focus on:

- Development of clusters
- Changes in trend
- Geographical spread and jump diffusion
- Detection of initiation of epidemics

> This has a huge impact on how we go about modeling



# BAYESIAN MODELING OF SPATIO-TEMPORAL HEALTH DATA

Count outcome in m small areas

$$\{\mathcal{Y}_i\}_{i=1,2,\ldots,m}$$

Poisson likelihood model

$$y_i \sim Po(\mu_i = e_i \, \theta_i)$$

e<sub>i</sub>: expected count of disease representing the background population effect (fixed)

 $\theta_i$ : unknown area-specific relative risk (focus of study)

# BAYESIAN MODELING OF SPATIO-TEMPORAL HEALTH DATA

Simple estimate of the relative risk: standardized incidence ratio (SIR), defined as the ratio of observed to expected counts

$$\hat{\theta}_i = y_i / e_i$$

This is a crude estimator and sometimes difficult to interpret and unstable

> We can assign a prior on  $\theta_i$  or we can model its logarithm. The data likelihood forms a hierarchy with the parameter priors to give a hierarchical model (Bayesian hierarchical model)

### **RELATIVE RISK MODELS**

 $log(\mu_i) = log(e_i) + log(\theta_i)$  $log(\theta_i) = ....model terms$ 

- A) Intercept (constant) model
- B) Log-normal (random intercept) model
- C) GLMM
- D) Convolution model

### **RELATIVE RISK MODELS**

D) Convolution model: Special case of GLMM that includes spatial correlation

$$\log(\theta_i) = \rho + u_i + v_i$$

 $\rho$ : overall level of the relative risk

convolution

 $\boldsymbol{u}_{i^{\text{i}}}$  spatially structured effect

 $\mathbf{v}_i\!\!:$  spatially unstructured extra variation

Adding covariates is straightforward:

$$\log(\theta_i) = \rho + \alpha_1 x_{1i} + u_i + v_i$$

### **RELATIVE RISK MODELS**

The improper CAR model

$$u_{i} \mid u_{j \neq i} \sim N\left(\frac{1}{\mid n_{i} \mid} \sum_{j \in n_{i}} u_{j}, \frac{\sigma_{u}^{2}}{\mid n_{i} \mid}\right)$$

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## MODEL FITTING: MCMC AND INLA

Conventionally Markov chain Monte Carlo is used to estimate posterior quantities for Bayesian models (such as the convolution or log-normal models)

- > WinBUGS is designed to do this via two basic methods
  - Gibbs sampling
  - > Metropolis –Hastings

Approximation of posterior distributions has recently become available via Laplace approximation in the INLA package

- > Does not require iterative computation (unlike McMC)
- Fast computation

# **PROSPECTIVE FITTING ISSUES**

#### Refitting at each new time point?

- Could be computationally poor
- Could use surveillance residuals

#### > Evolving model fitting

- Endemic-epidemic approach
- Particle filtering
  - Resampling parameter values given new data

(Lawson and Kleinman, 2005, ch 4, ch 5)

# **OUR WORK**

#### **BAYESIAN DISEASE SURVEILLANCE**





Posterior  $p(A|B) = \frac{p(B|A)p(A)}{p(A)}$ 

Prior Likelihood probability p(B)

### **OBJECTIVE**





Monthly counts of Salmonellosis cases in SC (1995-2003)

### HOW?

By using a model-based surveillance technique that incorporates both temporal and spatial information

Idea: Use a statistical model to describe the overall behavior of disease in space and time under 'normal' conditions

and

detect unusual departures from predictable patterns based on the estimated model

### **ADVANTAGES**

> Models: - allow covariate effects to be estimated

- provide insight into etiology, spread, prediction and control of disease

> The use of spatial information increases the power to detect small localized outbreaks of disease



Spatial distribution of the SMR from August to October 1996

# OUTLINE

#### > Univariate scenario

- > Model: The convolution model
- Surveillance technique: SCPO
- Case study: Salmonellosis

#### > Multivariate scenario

- > Model: The shared component model
- Surveillance technique: MSCPO
- Case study: ERD for respiratory diseases

#### > Can we go one step forward and anticipate disease outbreaks?

Syndromic information

> Monitor a map of m small areas over T time periods

$$\{y_{it}\}\ i=1,2,...,m;t=1,2,...,T$$

Bayesian hierarchical Poisson count model

$$y_{it} \sim Po(e_{it} \, \theta_{it})$$

e<sub>it</sub>: expected counts of disease (background population effect)

 $\theta_{it}$ : unknown area-specific relative risks

Convolution model (Besag et al., 1991, Lawson, 2013) vs

 $\log(\theta_{it}) = \rho + u_i + v_i$ 

ho: overall level of the relative risk;  $ho \sim N(0, \sigma_
ho^2)$ 

u<sub>i</sub>: spatially structured extra variation (improper CAR)

$$u_i \mid u_{j \neq i} \sim N\left(\frac{1}{\mid n_i \mid} \sum_{j \in n_i} u_j, \frac{\sigma_u^2}{\mid n_i \mid}\right)$$

 $v_i$ : spatially unstructured extra variation;  $v_i \sim N(0, \sigma_v^2)$  $\delta_{it}$ : space-time interaction random effect;  $\delta_{it} \sim N(0, \sigma_\delta^2)$ 

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Spatio-temporal model (knorr-Held, 2000)

$$\log(\theta_{it}) = \rho + u_i + v_i + \delta_{it}$$

> We ran a simulation scenario (details in Corberán-Vallet and Lawson, 2011) to compare both models

In terms of sensitivity, specificity and median time to detection, the convolution model outperformed the spatio-temporal model

In a surveillance context:

- > The model must describe the behavior of disease under endemic conditions
- It must be sensitive to temporal changes in the RR pattern of disease. A too complex model may absorb changes in risk in the model fit

> For seasonal data, in order to detect counts of disease higher than expected:

$$\log(\theta_{it}) = \rho + u_i + v_i + \sum_{s=1}^{12} \alpha_s I_s(t)$$

 $\alpha_s$ : seasonal effects

I<sub>s</sub>(t): indicator function that takes the value 1 if time t corresponds to month s

> Different risks to account for seasonality, but the risks so defined are constant over time

# UNIVARIATE SCENARIO: SURVEILLANCE TECHNIQUE

Surveillance Conditional Predictive Ordinate (Corberán-Vallet and Lawson, 2011)

$$SCPO_{it} = f(y_{it} | y_{1:t-1}) = \int f(y_{it} | \theta_i, y_{1:t-1}) \pi(\theta_i | y_{1:t-1}) d\theta_i$$
  
$$\approx \frac{1}{J} \sum_{j=1}^{J} Po(y_{it} | e_{it} \theta_i^{(j)})$$

$$\left\{\boldsymbol{\theta}_{i}^{(j)}\right\}_{j=1}^{J} \sim \boldsymbol{\pi}(\boldsymbol{\theta}_{i} \mid \boldsymbol{y}_{1:t-1})$$

If  $SCPO_{it} < \alpha$   $\implies$  signal an alarm for area i

\* See also: Surveillance Kullback
Liebler divergence (SKL)
(Rotejanaprasert and Lawson,
2016). Extension of the SCPO and
behaves differently.



Monthly counts of Salmonellosis cases in SC (1995-2003)

Number of counties signaling an alarm at each time point during the surveillance period (1996-2003). Data for year 1995 used to estimate the model.

Decision rule SCPO < 0.08

	J	F	Μ	А	Μ	J	J	А	S	0	Ν	D
1996	1	0	2	2	1	1	0	3	3	5	2	3
1997	0	3	2	2	4	2	5	2	0	1	0	1
1998	2	0	0	3	1	1	3	3	3	2	2	3
1999	0	1	2	2	1	1	3	2	0	0	2	0
2000	2	1	3	0	1	5	2	1	1	1	3	1
2001	2	5	3	2	1	1	1	2	1	1	0	1
2002	1	1	1	2	0	1	3	6	4	5	5	2
2003	1	0	0	0	2	2	1	2	4	3	0	2







Spatial distribution of the SMR from August to October 1996

Greenville County



Temporal plots for Greenville and Spartanburg counties

Red points represent alarms



## **MULTIVARIATE SCENARIO**

> Surveillance systems are often focused on more than one disease within a predefined area

A common approach is to monitor each disease separately: any correlation between diseases is ignored

> We present a multivariate extension of the proposed surveillance technique that

> allows for correlation between diseases

> can detect outbreaks happening in either one or a combination of diseases

A possibility to jointly model the endemic behavior of the multiple diseases is the shared component model (knorr-Held and Best, 2001)

For the joint analysis of  $k \ge 2$  diseases, Held el al. (2005) proposed a generalized SCM (only spatial information)

Our shared component model formulation:

$$y_{itk} \sim Po(e_{itk} \theta_{ik})$$
$$\log(\theta_{ik}) = \rho_k + \sum_{l=1}^{L} \phi_{l,k} \delta_{l,k} w_{l,i} + \psi_{ik}$$

 $\rho_k$ : overall risk for disease k

L: number of spatial fields (CAR components)  $w_l = (w_{l,1}, w_{l,2}, ..., w_{l,m})$  $\varphi_{l,k} = 1$  if  $w_l$  has an influence on disease k, and  $\varphi_{l,k} = 0$  otherwise  $\delta_{l,k}$ : weight  $\psi_{ik}$ : spatial unstructured extra variation for disease k

Advantage: By using indicator variables, we do not have to specify the structure of the model in advance

### **MULTIVARIATE SCENARIO: SURVEILLANCE** TECHNIQUE

For each small area *i* and time period *t* 



 $\theta_{ik}$  : posterior relative risk estimated at the previous time period (data up to time t-1)

 $(y_{itk_1} \quad y_{itk_2} \quad \dots \quad y_{itk_r})$ 

counts higher than expected counts smaller than expected

# MULTIVARIATE SCENARIO: SURVEILLANCE TECHNIQUE

A multivariate extension of the surveillance conditional predictive ordinate can be defined as (Corberán-Vallet, 2012)

$$\begin{split} MSCPO_{it} &= f(y_{itk_{1}}, y_{itk_{2}}, \dots, y_{itk_{r}} \mid y_{1:t-1}) \\ &= \iint \dots \int f(y_{itk_{1}}, y_{itk_{2}}, \dots, y_{itk_{r}} \mid \theta_{ik_{1}} \mid \theta_{ik_{2}} \dots \mid \theta_{ik_{r}}) x \\ &\qquad \pi(\theta_{ik_{1}}, \theta_{ik_{2}}, \dots, \mid \theta_{ik_{r}} \mid y_{1:t-1}) \mid d\theta_{ik_{1}} \mid d\theta_{ik_{2}} \dots \mid d\theta_{ik_{r}} \\ &\approx \frac{1}{J} \sum_{j=1}^{J} Po(y_{itk_{1}} \mid e_{itk_{1}} \theta_{ik_{1}}^{(j)}) Po(y_{itk_{2}} \mid e_{itk_{2}} \theta_{ik_{2}}^{(j)}) \dots Po(y_{itk_{r}} \mid e_{itk_{r}} \theta_{ik_{r}}^{(j)}) \end{split}$$

 $\{\theta_{ik}^{(j)}\}_{j=1}^{J}$  set of RR sampled from the posterior distribution at time t-1

If  $MSCPO_{it} < \alpha$   $\longrightarrow$  signal an alarm for area i

Weekly emergency room discharges for respiratory diseases in South Carolina in 2009



- ➢ We confine our analysis to data collected from week beginning June 28 to week beginning December 27 (weeks 26 − 52 in previous figure)
- > 46 counties, 27 time periods, and 5 diseases
- > Expected counts (constant) are calculated using the data from the first 3 weeks
- > These data are also used to estimate the proposed SCM (we assume L = 10)
- > The estimated model contains 5 spatial fields



Structure of the estimated model

 $\log(\theta_{i1}) = \rho_{1} + \delta_{1,1} w_{1,i} + \delta_{2,1} w_{2,i} + \psi_{i1}$   $\log(\theta_{i2}) = \rho_{2} + \delta_{3,2} w_{3,i} + \psi_{i2}$   $\log(\theta_{i3}) = \rho_{3} + \delta_{1,3} w_{1,i} + \psi_{i3}$   $\log(\theta_{i4}) = \rho_{4} + \delta_{1,4} w_{1,i} + \delta_{4,4} w_{4,i} + \psi_{i4}$  $\log(\theta_{i5}) = \rho_{5} + \delta_{1,5} w_{1,i} + \delta_{5,5} w_{5,i} + \psi_{i5}$ 

Goodness of fit: DIC (pD) for the proposed shared component model and five independent convolution models

Model	Disease 1	Disease 2	Disease 3	Disease 4	Disease 5	Total
Proposed SCM	808.18	268.04	578.84	657.64	652.43	2965.13
	(39.85)	(20.06)	(32.07)	(33.22)	(32.27)	(157.46)
Convolution models	810.30	268.24	584.29	659.17	656.17	2978.16
	(40.80)	(19.92)	(33.88)	(33.86)	(32.62)	(161.08)

 $\succ$  For  $t = 4, 5, \dots, 27$ , the SCM is estimated using the data observed up to t-1

MSCPO values associated with the new data are analyzed to detect epidemic onsets

> An alarm for the *i* th county is sounded at time *t* if the MSCPO<sub>it</sub> < 0.05

Counts of disease detected as unusual are assumed to be missing when they become part of the history





<u>A comparison with the multivariate scan</u> <u>statistic</u>: Counties where an outbreak is declared

Left: Areas signaling an alarm based on the MSCPO

Right: Most likely cluster (MLC) and secondary clusters (SC) using the Poissonbased prospective space-time scan statistic

# MULTIVARIATE SCENARIO: A COMPARISON

> The space-time scan statistic pinpoints the general time and location of the most likely cluster (and possible secondary clusters)

- > Drawbacks:
  - Counties with no increased incidence can be included in the cluster
  - Some counties do not undergo an outbreak of disease for all the diseases reported in the cluster
  - Several large clusters covering practically all the study region are reported

> The MSCPO detects, at each time, counties with increased disease incidence and the diseases causing the alarm within each county

It enables a timelier and more informed response

# CAN WE ANTICIPATE DISEASE OUTBREAKS?

> We have developed a model-based surveillance technique to detect disease outbreaks as soon as possible

> But... can we predict disease outbreaks before they occur?

> The answer is based on the use of syndromic information

However, we do not want to monitor syndromes or health-related data that precede diagnosis (these data can lead to false alarms)

We want to develop a multivariate model that models both the disease of interest and syndromic information and helps to predict possible outbreaks

# CAN WE ANTICIPATE DISEASE OUTBREAKS? A FIRST ATTEMPT

The disease of interest is an infectious disease and we have information from a syndromic disease

 $y_{it} \sim Po(\mu_{it} + I_{it})$ 

We want a model like this for the infection of interest

endemic component: describes the pattern of disease during non-epidemic periods epidemic component: expected additive increase in disease counts due to an epidemic (depends on syndromic information)

# CAN WE ANTICIPATE DISEASE OUTBREAKS? A FIRST ATTEMPT

y<sub>it</sub>: number of cases of the disease of interest

 $y_{it} \sim Po(\mu_{it} + I_{it})$  $\mu_{it} = e_{it}\theta_{it}$  $\log(\theta_{it}) = \rho + u_i + v_i$ 

y<sub>it</sub><sup>s</sup>: number of cases of the syndromic disease

 $y_{it}^{s} \sim Po(\mu_{it}^{s} + I_{it}^{s})$  $\mu_{it}^{s} = e_{it}^{s} \theta_{it}^{s}$  $\log(\theta_{it}^{s}) = \rho^{s} + \psi u_{i} + v_{i}^{s}$ 

during non-epidemic conditions, the two diseases may be influenced by common confounding factors (Wang and Wall, 2003). Here  $\psi \sim N(0, \sigma_w^2)$ 

# CAN WE ANTICIPATE DISEASE OUTBREAKS? A FIRST ATTEMPT

y<sub>it</sub>: number of cases of the disease of interest

y<sub>it</sub><sup>s</sup>: number of cases of the syndromic disease

$$y_{it} \sim Po(\mu_{it} + I_{it}) \qquad \qquad y_{it}^{s} \sim Po(\mu_{it}^{s} + I_{it}^{s})$$

$$(I_{it}) = \beta_{it} \left( y_{i,t-1} + \gamma_{i} \sum_{j \in n_{i}} y_{j,t-1} \right) + \phi_{i} I_{i,t-1}^{s} \qquad \qquad I_{it}^{s} = \beta_{it}^{s} \left( y_{i,t-1}^{s} + \gamma_{i}^{s} \sum_{j \in n_{i}} y_{j,t-1}^{s} \right)$$

Component based on data up to time t-1. At time t we can make predictions for time t+1

# **CONCLUDING REMARKS**

> We have presented a Bayesian model-based surveillance technique for on-line spatio-temporal public health surveillance

As a local measure, different alarms are sounded for those areas of increased disease incidence

It can be applied in any surveillance context where a model is used to describe the endemic behavior of diseases

> Simple spatial models are the key to allowing detection of change over time

# **CONCLUDING REMARKS**

 $\succ$  The technique can be easily extended for the monitoring of multiple diseases

> The proposed SCM allows us to identify the number of latent spatial fields required to describe the correlation across both areas and diseases

> The multivariate surveillance technique improves outbreak detection when changes in disease incidence happen simultaneously

> Finally, we have presented a model that incorporates syndromic information to predict the start of epidemics

Some preliminary results obtained in a prospective analysis of infectious disease data showed its good performance

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